Six Field Governing Equations for RELAP5

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Overview

• New Governing Equations
• Closure Relationships
Two-Phase Methodology

- TRAC, VIBRE, RELAP5, RELAP5/SCDAP, RELAP5-3D & most of the system and subchannel codes have 2 fields:
  - liquid and vapor
- Control volumes are completely liquid, completely vapor, or partially liquid/vapor
  - Void fraction computed to determine percentage of control volume that is vapor
  - Lumped Approach is used
Flow Regimes - Theoretical -

Simple Pipe Geometries
Flow Regimes - In Reality -

- Horizontal Flow

- Simple Pipe

- Fuel Core

- Fuel Bundle

- Flow Regimes

- College of Engineering
Six-Field Model

- Increase modeled fields in RELAP to include bubbles and droplets
- Mass, Momentum, and Energy Balance Equations are developed for:
  1. Continuous Vapor
  2. Large Bubble
  3. Small Bubble
  4. Continuous Liquid
  5. Large Droplet
  6. Small Droplet

VAPOR

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Considerations For Multiple Field Models

- **Multiple interfaces between fields**
  - Phase change
  - Shear forces
- **Closure relationships required**
  - Heat Transfer
  - Relative Velocities
- **Physical phenomena that cause field transitions**
  - Entrainment
  - De-entrainment
  - Spacer grids
  - Flow breakup
Mass Balance - Continuous Liquid

\[ \frac{\partial}{\partial t} (\alpha_f \rho_f) + \nabla \cdot (\alpha_f \rho_f \vec{v}_f) = -\Gamma_g \]

- **Time rate of change of mass**
- **Mass convection**
- **Phase change**

\[ S_{LD,E}''' - S_{SD,E}''' + S_{LD,DE}''' + S_{SD,DE}''' \]

- **Mass exchange due to entrainment/de-entainment**

- **\( S_{LD,E}''' \)** - Loss term for large droplet entrainment
- **\( S_{SD,E}''' \)** - Loss term for small droplet entrainment
- **\( S_{LD,DE}''' \)** - Source term for large droplet de-entrainment
- **\( S_{SD,DE}''' \)** - Loss term for small droplet de-entrainment
Mass Balance - Large Droplet

\[
\frac{\partial}{\partial t} (\alpha_{LD} \rho_f) + \nabla \cdot (\alpha_{LD} \rho_f \vec{v}_{LD}) = -\Gamma_{LD} + S_{LD,E}''' - S_{LD,DE}''' - S_{LD,SB}''' + S_{SD,C}'''
\]

- \( S_{LD,E}''' \) - Source term for large droplet entrainment (same as previous equation)
- \( S_{SD,C}''' \) - Source term from small droplets coalescing into large droplets
- \( S_{LD,DE}''' \) - Loss term for large droplet de-entrainment
- \( S_{LD,SB}''' \) - Loss term for large droplets breaking up on spacer grids (joining small droplet field)
- \( S_{LD,FB}''' \) - Loss term for large droplet flow break-up
Mass Balance - Small Droplet

\[
\frac{\partial}{\partial t} (\alpha_{SD} \rho_f) + \nabla \cdot (\alpha_{SD} \rho_f \vec{v}_{SD}) = -\Gamma_{SD} + S_{SD,E}''' + S_{LD,SB}''' + S_{LD,FB}''' - S_{SD,DE}''' - S_{SD,C}'''
\]

- \(S_{SD,E}'''\) - Source term for small droplet entrainment
- \(S_{LD,SB}'''\) - Source term from large droplets breaking up on spacer grids
- \(S_{LD,FB}'''\) - Source term for large droplet flow break-up
- \(S_{SD,DE}'''\) - Loss term for small droplet de-entrainment
- \(S_{SD,C}'''\) - Loss term from small droplets coalescing into large droplets
Momentum Balance - Continuous Liquid

\[ \alpha_f \rho_f \frac{D\bar{u}_f}{Dt} = -\alpha_f \nabla p_f + \nabla \cdot [\alpha_f (\mathcal{S}_f + \mathcal{S}^T_f)] + \]

- Rate of change of momentum
- Momentum change due to pressure gradient
- Average viscous stress and turbulent stress effects

Body force effects

\[ \alpha_f \rho_f g \bar{y}_f + (p_{fi} - p_f) \nabla \alpha_f + (\bar{v}_{i,L} - \bar{v}_f) \Gamma_L + (\bar{v}_{i,LB} - \bar{v}_f) \Gamma_{LB} + (\bar{v}_{i,SBu} - \bar{v}_f) \Gamma_{SBu} + \]

- Pressure change between interface and continuous liquid
- Momentum exchanged from phase change

Interfacial and skin drag

\[ \nabla \alpha_f \cdot \mathcal{S}_{fi,g} - \nabla \alpha_f \cdot \mathcal{S}_{fi,SBu} - \nabla \alpha_f \cdot \mathcal{S}_{fi,LB} - \]

- Momentum Transfer by Interfacial Shear

Droplet Entrainment/De-Entrainment

\[ S''_{LD,EU}v_{LD} - S''_{SD,EU}v_{SD} + S''_{SD,DE}v_{SD} + S''_{LD,DE}v_{LD} \]
Momentum Balance - Continuous Liquid Source Terms

\[ S''_{LD,E}v_{LD} - S'''_{SD,E}v_{SD} + S'_{{SD,DE}}v_{SD} + S''_{LD,DE}v_{LD} \]

Droplet Entrainment/De-Entrainment

- \( v_{LD}, v_{SD} \) - velocities of large and small droplets, respectively
- \( S''_{LD,E} \) - Loss term for large droplet entrainment
- \( S'''_{SD,E} \) - Loss term for small droplet entrainment
- \( S'_{{SD,DE}} \) - Source term for large droplet de-entrainment
- \( S''_{LD,DE} \) - Loss term for small droplet de-entrainment
Momentum Balance - Large Droplets

\[ \frac{D\bar{v}_{LD}}{Dt} = -\alpha_{LD} \nabla p_{LD} + \alpha_{LD} \rho_f \bar{g}_{LD} + (p_i,LD - p_{LD}) \nabla \alpha_{LD} + \\
(\bar{v}_{i,LD} - \bar{v}_{LD}) \Gamma_{LD} + M_{i,LD} - \nabla \alpha_{LD} \cdot \zeta_{LDi,g} + \\
S_{LD,E}'\bar{v}_{LD} - S_{LD,SB}'\bar{v}_{LD} - S_{LD,FB}'\bar{v}_{LD} - S_{LD,DE}'\bar{v}_{LD} + S_{SD,C}'\bar{v}_{SD} \]

- **S_{LD,E}''''** - Source term for large droplet entrainment \(S_{SD,C}''''\)
- **S_{LD,DE}''''** - Loss term for large droplet de-entrainment
- **S_{LD,SB}''''** - Loss term for large droplets breaking up on spacer grids (joining small droplet field)
- **S_{LD,FB}''''** - Loss term for large droplet flow break-up
Momentum Balance - Small Droplets

\[ \alpha_{SD} \rho_f \frac{D\vec{v}_{SD}}{Dt} = -\alpha_{SD} \nabla p_{SD} + \alpha_{SD} \rho_f \vec{g}_{SD} + (p_{i,SD} - p_{SD}) \nabla \alpha_{SD} + \]

\[ (\vec{v}_{i,SD} - \vec{v}_{SD}) \Gamma_{SD} + M_{i,SD} - \nabla \alpha_{SD} \cdot \vec{z}_{SDi,g} + \]

\[ S''_{SD,E} \vec{v}_{SD} + S''_{LD,SB} \vec{u}_{LD} + S''_{LD,FB} \vec{u}_{LD} - S''_{SD,DE} \vec{v}_{SD} - S''_{SD,C} \vec{v}_{SD} \]

- \( S''_{SD,E} \) - Source term for small droplet entrainment
- \( S''_{LD,SB} \) - Source term from large droplets breaking up on spacer grids
- \( S''_{LD,FB} \) - Source term for large droplet flow break-up
- \( S''_{SD,DE} \) - Loss term for small droplet de-entrainment
- \( S''_{SD,C} \) - Loss term from small droplets coalescing into large droplets
Energy Balance - Continuous Liquid

\[
\alpha_f \rho_f \frac{D_f h_f}{Dt} = -\nabla \cdot \alpha_f (\vec{q}_f - \vec{q}_f^T) + \alpha_f \frac{D_f p_f}{Dt} + \Phi_f^T + \Phi_f^n + 
\]

Rate of energy change, with convective effects

Average conduction and turbulent heat flux

Flow work

Turbulent work effect source and viscous dissipation

\[
\Gamma_f,i (h_{f,i} - h_f) + \Gamma_{f,w} (h_{f,w} - |h_f|) + \Gamma_{f,SBu} (h_{f,SBu} - h_f) + \Gamma_{f,LB} (h_{f,LB} - h_f) + 
\]

Energy exchange due to phase change at interfaces and near the wall

\[
a_{i,i} q_{f,i}''' + a_{i,SBu} q_{SBu,i}''' + a_{i,LB} q_{LB,i}''' + a_{w,f} q_{w,f}''' + 
\]

Energy exchange due to heat transfer at interfaces and from the wall

\[
(p_f - p_{f,i}) \frac{D_f \alpha_f}{Dt} + M_{i,f} \cdot (\vec{v}_{f,i} - \vec{v}_f) - \nabla \alpha_f \cdot \zeta_{f,i} (\vec{v}_{f,i} - \vec{v}_f) - 
\]

Interfacial pressure differences

Interfacial drag between continuous fields

Interfacial shear stress

\[
S_{LD,E} h_f - S_{SD,E} h_f + S_{LD,DE} h_{LD} + S_{SD,DE} h_{SD} 
\]

Entrainment/de-entrainment
Energy Balance - Continuous Liquid Source Terms

\[ S''''_{LD,E} h_f - S''''_{SD,E} h_f + S''''_{LD,DE} h_{LD} + S''''_{SD,DE} h_{SD} \]

- \( h_f \) - Enthalpy of continuous liquid
- \( h_{LD}, h_{SD} \) - Enthalpy of large and small droplets
- \( S_{LD,E} '''' \) - Loss term for large droplet entrainment
- \( S_{SD,E} '''' \) - Loss term for small droplet entrainment
- \( S_{LD,DE} '''' \) - Source term for large droplet de-entrainment
- \( S_{SD,DE} '''' \) - Loss term for small droplet de-entrainment
Energy Balance - Large Droplet

\[
\alpha_{LD}\rho_{LD}\frac{D_{LD}h_{LD}}{Dt} = \Gamma_{LD,i} \left( h_{LD,i} - h_{LD} \right) + a_i q'''
\]

\[
(p_{LD} - p_{LD,i}) \frac{D_{LD}\alpha_{LD}}{Dt} + M_{i,LD} \cdot (\vec{v}_{LD,i} - \vec{v}_{LD}) - \nabla \alpha_{LD} \cdot \vec{S}_{LD,i} \cdot (\vec{v}_{LD,i} - \vec{v}_{LD}) + S''''_{LD,C} h_{SD} + S''''_{LD,E} h_f - 
\]

\[
S''''_{LD,SB} h_{LD} - S''''_{LD,FB} h_{LD} - S''''_{LD,DE} h_{LD}
\]
Energy Balance - Small Droplet

\[ \alpha_{SD} \rho_{SD} \frac{D_{SD} h_{SD}}{D t} = \Gamma_{SD,i} (h_{SD,i} - h_{SD}) + a_{i} q_{SD,i}'''' + (p_{SD} - p_{SD,i}) \frac{D_{SD} \alpha_{SD}}{D t} + M_{i,SD} \cdot (\nu_{SD,i} - \nu_{SD}) - \nabla \alpha_{SD} \cdot \nabla_{SD,i} (\nu_{SD,i} - \nu_{SD}) - S''''_{LD,c} h_{SD} + S''''_{SD,E} h_{f} + S''''_{LD,SB} h_{LD} + S''''_{LD,FB} h_{LD} - S''''_{SD,DE} h_{SD} \]
Governing Equation Closure

- Source terms must be resolved to solve governing equations
- Source term solutions depend on flow regime
- Flow regimes are determined in RELAP
  - Flow rate
  - Subcooling
  - Void fraction
Flow Regime Determination

- Flow regime determined by void fraction, flow velocity, subcooling, and orientation
- Regime determines heat transfer correlations between phases and pipe walls
Flow Regime Determination

• Same maps used for channel and pipe flows
  – Individual correlations for channels and pipes
Mass Closure

- Physical transport of working fluid between fields
- Model of droplet breakup on spacer grids
- Model of flow breakup
  - Based on Weber number
- Droplet entrainment
  - Orientation-dependent
- Models to capture coalescence of small droplets
- Entrainment/De-Entrainment models
- The bulk of these models will be new to RELAP
Momentum Closure

- Relative velocities between fields
  - Interfacial drag used to compute relative velocity
    - Drift Flux Model
    - Drag Coefficient Model
    - Depend on flow configuration and characteristics
  - Wall drag also impacts field velocities
- Six-Field equations allow for specifics of flow geometry to be used in drag calculations
Energy Closure

- Heat transfer between fields needed to compute relative enthalpy of each field
- Models already available in RELAP for heat transfer between bubbles and vapor, annular flow and vapor core, etc.
- Six-Field equations allow for specifics of flow geometry to be used in heat transfer calculations
Selected Relevant Publications

- **Derivation of new mass, momentum, and energy conservation equations for two-phase flows**, GA Roth, F Aydogan, Progress in Nuclear Energy 80, 90-101
- **Six-Field Governing Equation Development for Advanced System Codes**, G Roth, F Aydogan, Nureth-16, 2015
- **Momentum and Energy Closure Models for Two-Phase Flow Six-Field Model**, G Roth, F Aydogan, Nuclear Engineering and Design Journal, Under Review
- **Mass Closure Models for Two-Phase Flow Six-Field Model**, G Roth, F Aydogan, Nuclear Engineering and Design, Under Review
Future Work

• Implement governing equations and closure models in RELAP5