Variable Gravity

Dr. George Mesina

RELAP5 International Users Seminar

Idaho Falls, ID

September 11-12, 2014

INL/CON-14-32950

Outline

- **Uses**
- **Theory**
- **Description of Motion**
- **Input**

Uses for Variable Gravity

- The gravitational constant can be input to represent reactors/systems in non-accelerating conditions
  - Lunar or a planetary w/ non-earth gravity
  - Space station
  - Probes and spacecraft outside Earth orbit
- Additional changes are required to represent systems in a non-inertial frame of reference
  - Earthquake scenarios
  - Aircraft and watercraft
  - Off-shore power plants (E.G. MIT floating reactor)

Moving System Theory

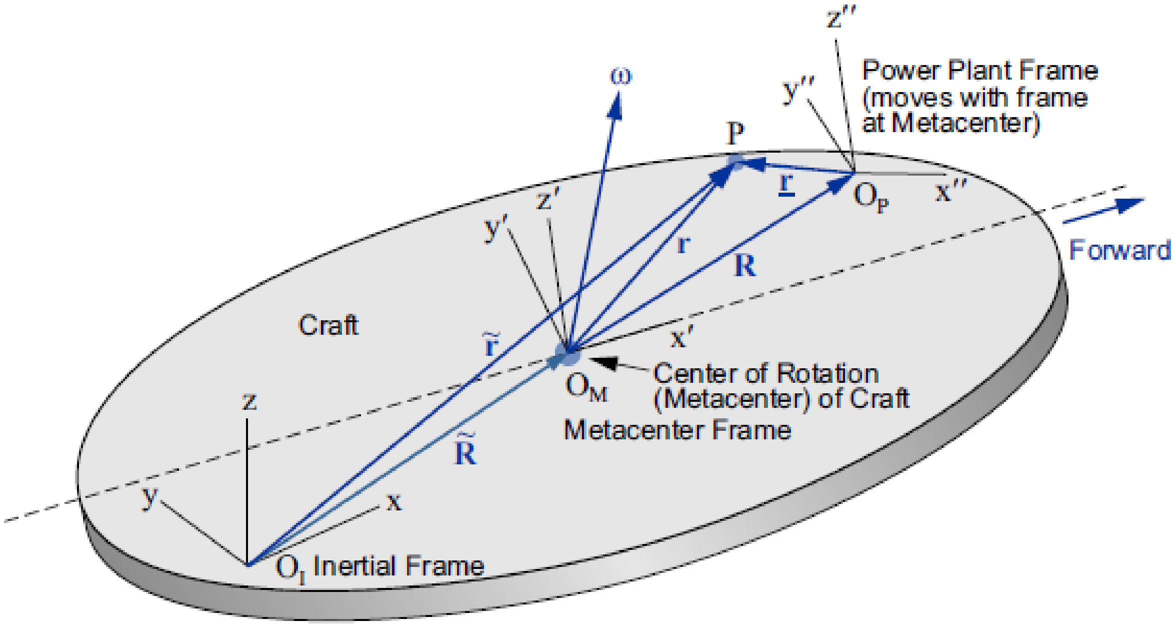
- "Craft" has rotational and translational acceleration
- No effect on continuity or energy equation
- Accelerations affect momentum equation
  - They are added to gravitational acceleration to obtain an "effective gravity" vector

$$\frac{\partial \rho v}{\partial t} + (v \cdot \nabla)\rho v = -\nabla P + \nabla \cdot [\![\tau]\!] + \rho(g - a_{acc})$$

P = pressure
V = velocity
R = density
$[\![\tau]\!]$ = shear stress tensor

Need to express acceleration vector in terms of translating and rotating reference frame

Moving System Theory



- **Metacenter frame is attached to the center of the craft**
- **It is a non-inertial frame**
- **a_{acc} is the translational & rotational acceleration of the <u>Metacenter frame</u>**

Moving System Theory

Metacenter Axes

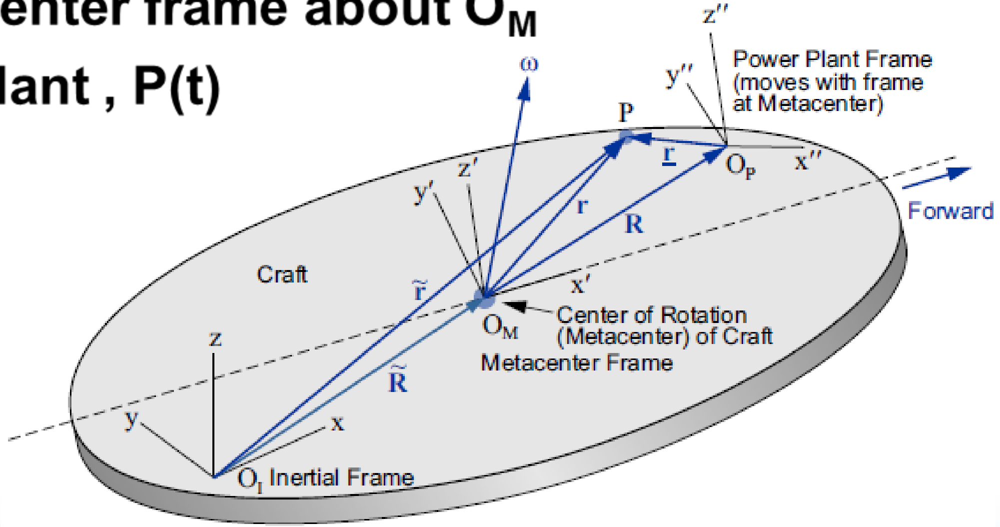
- **x'-axis: along craft's direction of motion <u>forward is positive</u>**
- **y'-axis: transverse to x-axis. Facing forward, <u>left is positive</u>**
- **z'-axis: vertical of craft: <u>up is positive</u>**

ω = Rotation vector of Metacenter frame about O_M

Position of point on power plant , P(t)

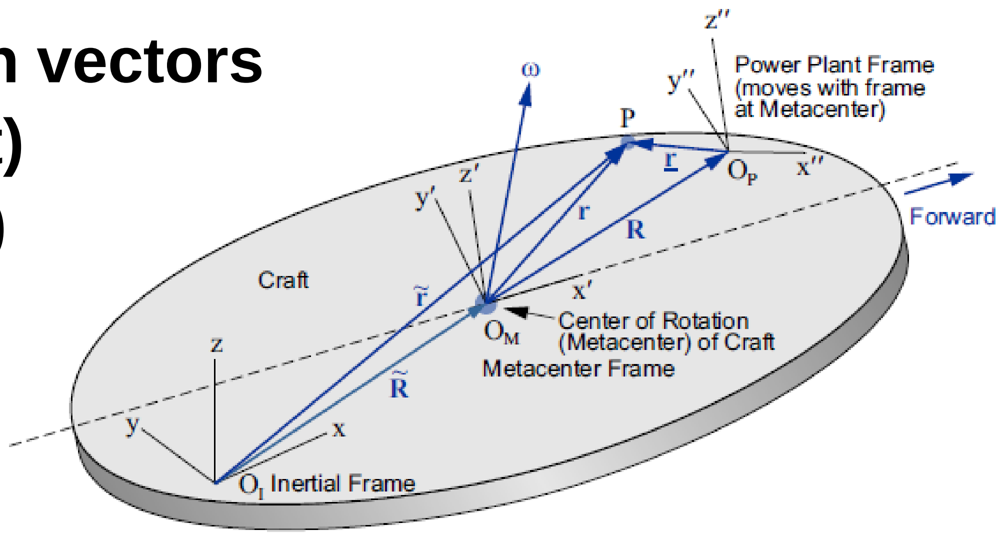
Relative position vectors

- **Relative (to O_P), $\underline{r}(t)$**
- **Relative (to O_M), r(t)**
- **Relative (to O_I), $\tilde{r}(t)$**



Relate Inertial & Metacenter Velocity

- **Relative center-position vectors**
  - **O_P relative to O_M, R(t)**
  - **O_M relative to O_I, $\widetilde{R}$(t)**
- **Inertial frame**
  - $\tilde{r}(t) = \widetilde{R}(t) + r(t)$
  - $\tilde{r}(t) = \frac{d\widetilde{R}}{dt} + \frac{dr}{dt}$
- **Metacenter frame**
  - **Unit direction vectors i', j', k' in x'-, y'-, z'-directions**
  - $r(t) = x'i' + y'j' + z'k'$
  - $\frac{dr}{dt} = \frac{d}{dt}[x'i' + y'j' + z'k']$



Inertial Time-derivative of Metacenter Position Vector, r

Product Rule to $\frac{d\boldsymbol{r}}{dt} = \frac{d}{dt}[x'\boldsymbol{i}' + y'\boldsymbol{j}' + z'\boldsymbol{k}']$

- $\frac{d\boldsymbol{r}}{dt} = \boxed{\frac{dx'}{dt}\boldsymbol{i}' + \frac{dx'}{dt}\boldsymbol{j}' + \frac{dx'}{dt}\boldsymbol{k}'} + \boxed{x'\frac{d\boldsymbol{i}'}{dt} + y'\frac{d\boldsymbol{j}'}{dt} + z'\frac{d\boldsymbol{k}'}{dt}}$

But the velocity, v, of P(t) in the *Metacenter* frame is:

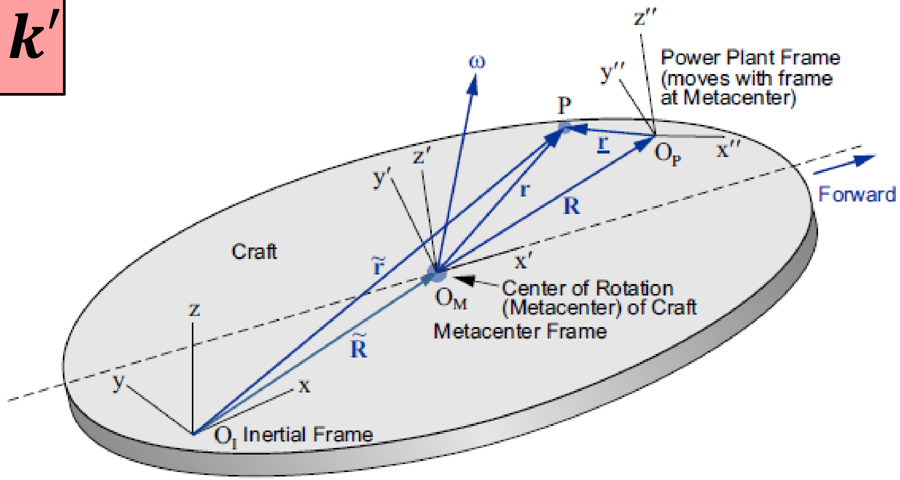
- $\boldsymbol{v}(t) = \frac{D\boldsymbol{r}}{Dt} = \boxed{\frac{dx'}{dt}\boldsymbol{i}' + \frac{dx'}{dt}\boldsymbol{j}' + \frac{dx'}{dt}\boldsymbol{k}'}$

From other sources:

- $\boldsymbol{\omega} \times \boldsymbol{r} = \boxed{x'\frac{d\boldsymbol{i}'}{dt} + y'\frac{d\boldsymbol{j}'}{dt} + z'\frac{d\boldsymbol{k}'}{dt}}$

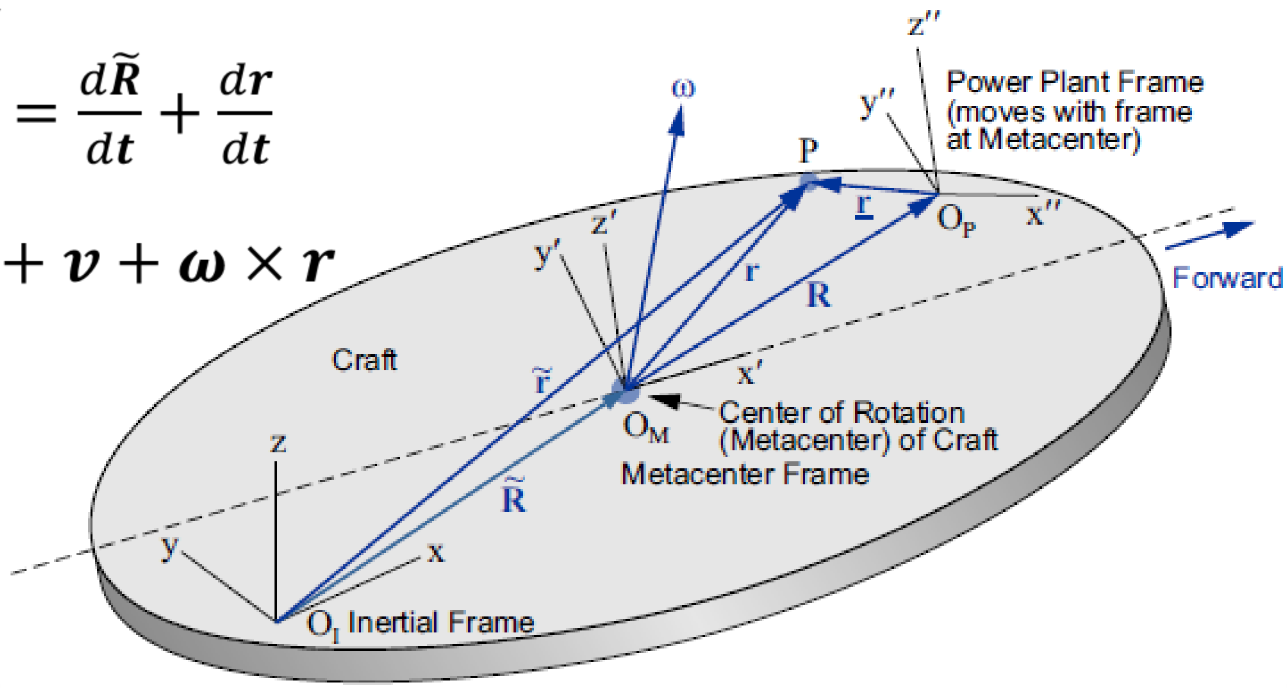
THEOREM

- $\frac{d\boldsymbol{r}}{dt} = \frac{D\boldsymbol{r}}{Dt} + \boldsymbol{\omega} \times \boldsymbol{r} = \boldsymbol{v} + \boldsymbol{\omega} \times \boldsymbol{r}$



Inertial & Metacenter Velocity Relationship

- **Substitute:** $\frac{d\boldsymbol{r}}{dt} = \boldsymbol{v} + \boldsymbol{\omega} \times \boldsymbol{r}$
- **Into:** $\widetilde{\boldsymbol{v}}(\boldsymbol{t}) = \frac{d\tilde{\boldsymbol{r}}}{dt} = \frac{d\widetilde{\boldsymbol{R}}}{dt} + \frac{d\boldsymbol{r}}{dt}$
- $\widetilde{\boldsymbol{v}}(\boldsymbol{t}) = \frac{d\tilde{\boldsymbol{r}}}{dt} = \frac{d\widetilde{\boldsymbol{R}}}{dt} + \boldsymbol{v} + \boldsymbol{\omega} \times \boldsymbol{r}$



Idaho National Laboratory

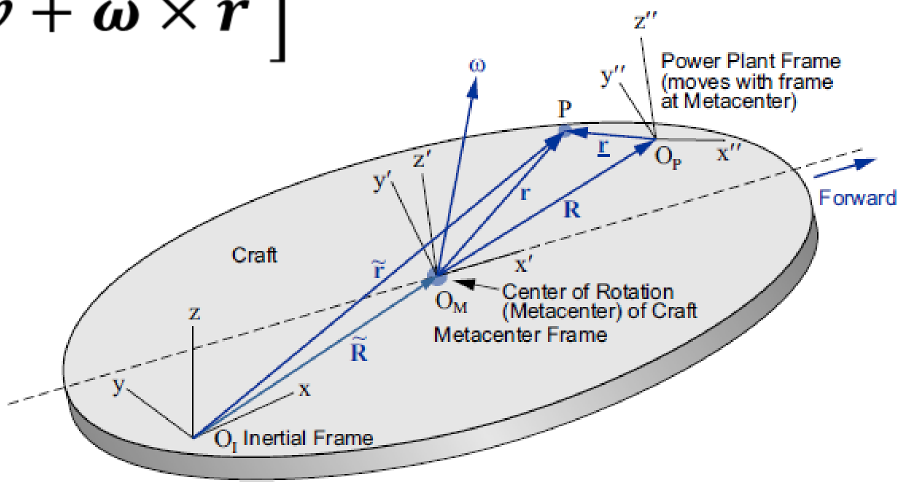
Relate Inertial & Metacenter Acc.

Inertial: $\tilde{\boldsymbol{a}}(\boldsymbol{t}) = \frac{d\tilde{\boldsymbol{v}}}{dt}$

- $\tilde{\boldsymbol{a}}(\boldsymbol{t}) = \frac{d}{dt}\left[\frac{d\tilde{\boldsymbol{R}}}{dt} + \frac{d\boldsymbol{r}}{dt}\right] = \frac{d}{dt}\left[\frac{d\tilde{\boldsymbol{R}}}{dt} + \boldsymbol{v} + \boldsymbol{\omega}\times\boldsymbol{r}\right]$
- $\tilde{\boldsymbol{a}}(\boldsymbol{t}) = \left[\frac{d^2\tilde{\boldsymbol{R}}}{dt^2} + \boxed{\frac{d\boldsymbol{v}}{dt}} + \boxed{\frac{d(\boldsymbol{\omega}\times\boldsymbol{r})}{dt}}\right]$

Metacenter: $\boldsymbol{a} = \frac{D\boldsymbol{v}}{Dt}$

- **From** $\boxed{\textbf{THM: } \frac{d\boldsymbol{r}}{dt} = \frac{D\boldsymbol{r}}{Dt} + \boldsymbol{\omega}\times\boldsymbol{r}}$
  - $\boxed{\frac{d\boldsymbol{v}}{dt}} = \frac{D\boldsymbol{v}}{Dt} + \boldsymbol{\omega}\times\boldsymbol{v} = \boldsymbol{a} + \boldsymbol{\omega}\times\boldsymbol{v}$
  - $\boxed{\frac{d(\boldsymbol{\omega}\times\boldsymbol{r})}{dt}} = \frac{D(\boldsymbol{\omega}\times\boldsymbol{r})}{Dt} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r})$



Relate Inertial & Metacenter Acc.

Apply the product rule to $\frac{D(\boldsymbol{\omega}\times\boldsymbol{r})}{Dt}$

- $\frac{D(\boldsymbol{\omega}\times\boldsymbol{r})}{Dt} = \frac{D\boldsymbol{\omega}}{Dt}\times\boldsymbol{r} + \boldsymbol{\omega}\times\frac{D\boldsymbol{r}}{Dt} = \frac{D\boldsymbol{\omega}}{Dt}\times\boldsymbol{r} + \boldsymbol{\omega}\times\boldsymbol{v}$

Therefore

- $\widetilde{\boldsymbol{a}}(\boldsymbol{t}) = \left[\frac{d^2\widetilde{\boldsymbol{R}}}{dt^2} + \frac{d\boldsymbol{v}}{dt} + \frac{d(\boldsymbol{\omega}\times\boldsymbol{r})}{dt}\right]$
- $\widetilde{\boldsymbol{a}}(\boldsymbol{t}) = \frac{d^2\widetilde{\boldsymbol{R}}}{dt^2} + \boldsymbol{a} + \boldsymbol{\omega}\times\boldsymbol{v} + \frac{D(\boldsymbol{\omega}\times\boldsymbol{r})}{Dt} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r})$
- $\widetilde{\boldsymbol{a}}(\boldsymbol{t}) = \frac{d^2\widetilde{\boldsymbol{R}}}{dt^2} + \boldsymbol{a} + \boldsymbol{\omega}\times\boldsymbol{v} + \frac{D\boldsymbol{\omega}}{Dt}\times\boldsymbol{r} + \boldsymbol{\omega}\times\boldsymbol{v} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r})$

Relate Inertial & Metacenter Acc.

- $\tilde{\boldsymbol{a}}(\boldsymbol{t}) = \frac{\boldsymbol{d^2\tilde{R}}}{\boldsymbol{dt^2}} + \boldsymbol{a} + \boldsymbol{2\omega} \times \boldsymbol{v} + \frac{\boldsymbol{D\omega}}{\boldsymbol{Dt}} \times \boldsymbol{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$ **Eq. A**
- **Denote translational motions surge, drift and heave by $\ddot{x}, \ddot{y}$ *and* $\ddot{z}$**
    - $\frac{\boldsymbol{d^2\tilde{R}}}{\boldsymbol{dt^2}} = \ddot{\boldsymbol{x}}\boldsymbol{i} + \ddot{\boldsymbol{y}}\boldsymbol{j} + \ddot{\boldsymbol{z}}\boldsymbol{k}$
- **Within the craft's moving frame, a=0**
- **If $\boldsymbol{v}$ and $\boldsymbol{\omega}$ are given, Eq. A is a set of 3 coupled, 2^{nd} order, linear ordinary differential equations**
    - **Solving this gives acceleration of the moving particle**
    - **In the momentum equation:** $\boldsymbol{a_{acc}} = \tilde{\boldsymbol{a}}(\boldsymbol{t}) - \boldsymbol{a}$

Idaho National Laboratory

Functions of Time

- Translation directions are functions of time
- The rotation vector angles are also functions of time
- RELAP5-3D allows two sets of angle data for rotation
  - Pitch-yaw-roll : (γ, β, α)
  - Euler angles: (φ, θ, Ψ)

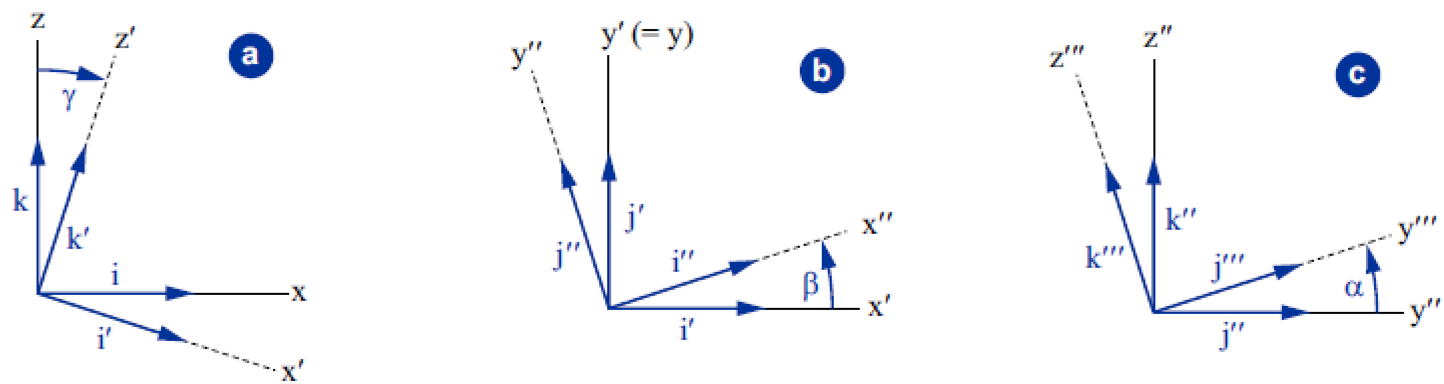
Idaho National Laboratory

Pitch-Yaw-Roll Angle Specification

a) <u>Pitch</u>: angular displacement γ about the inertial y-axis
   – Pitch forward (and back)

b) <u>Yaw</u>: angular displacement β about the new z-axis (z'-axis)
   – Twist (like a bottle cap)

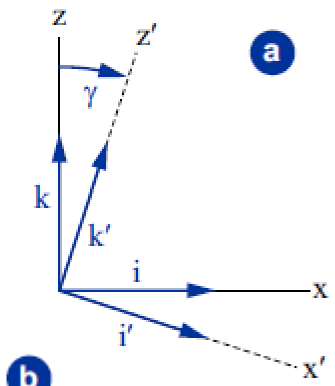
c) <u>Roll</u>: angular displacement α about metacenter x-axis (x''-axis)
   – Roll like a barrel (about longitudinal axis)



Pitch-Yaw-Roll Angle Specification

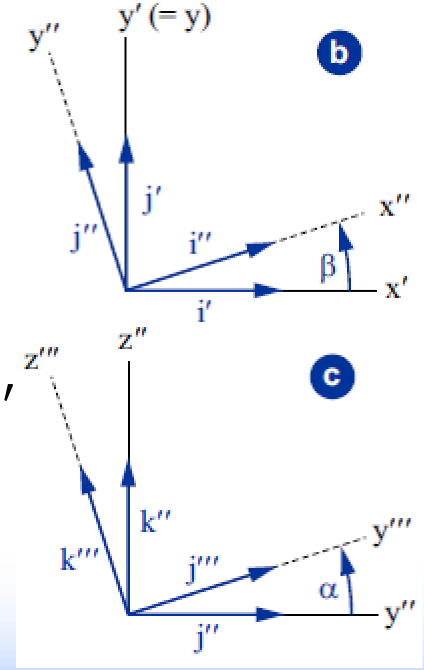
Elementary Rotation Matrices & coordinates

$$E_P = \begin{bmatrix} cos\gamma & 0 & -sin\gamma \\ 0 & 1 & 0 \\ sin\gamma & 0 & cos\gamma \end{bmatrix}, X' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = E_P X = E_P \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



$$E_Y = \begin{bmatrix} cos\beta & sin\beta & 0 \\ -sin\beta & cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}, X'' = \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = E_Y X'$$

$$E_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & sin\alpha \\ 0 & -sin\alpha & cos\alpha \end{bmatrix}, X''' = \begin{bmatrix} x''' \\ y''' \\ z''' \end{bmatrix} = E_R X''$$



- $X''' = E_R E_Y E_P X$ ***and*** $X = [E_R E_Y E_P]^T X'''$

Euler Angle Specification

- 1st Euler angle, φ, rotates about the inertial z-axis

$$E_\varphi = \begin{bmatrix} cos\varphi & sin\varphi & 0 \\ -sin\varphi & cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}, X' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = E_\varphi X = E_\varphi \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- 2nd Euler angle, θ, rotates about the new x'-axis

$$E_\theta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & sin\theta \\ 0 & -sin\theta & cos\theta \end{bmatrix}, X'' = \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = E_\theta X'$$

- 3rd Euler angle, ψ, rotates about the new z''-axis

$$E_\psi = \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, X''' = \begin{bmatrix} x''' \\ y''' \\ z''' \end{bmatrix} = E_\psi X''$$

- $X''' = E_\psi E_\theta E_\varphi X$ ***and*** $X = [E_\psi E_\theta E_\varphi]^T X'''$

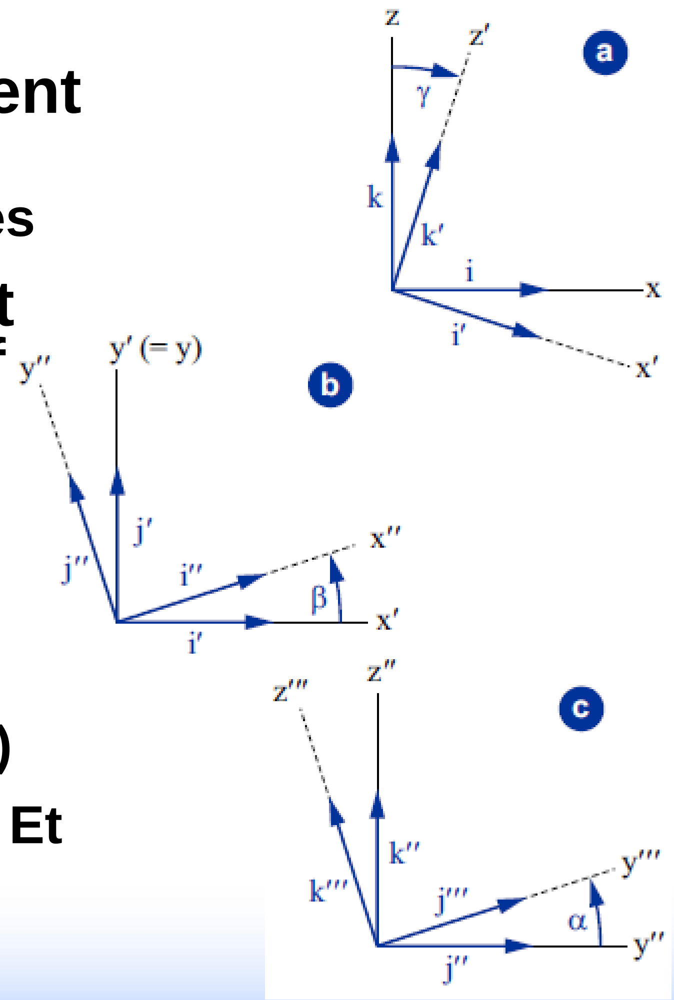
Idaho National Laboratory

Input: Fixed/Moving Option

- **RELAP5-3D can be used for a system fixed in space or a system that rotates and/or translates**
- **For <u>stationary systems</u>, Word 2 on Card 119 is entered as FIXED**
  - **If this word is not entered, a fixed problem is assumed**
- **For <u>systems that rotate and/or translate</u>, Word 2 on Card 119 is MOVING**

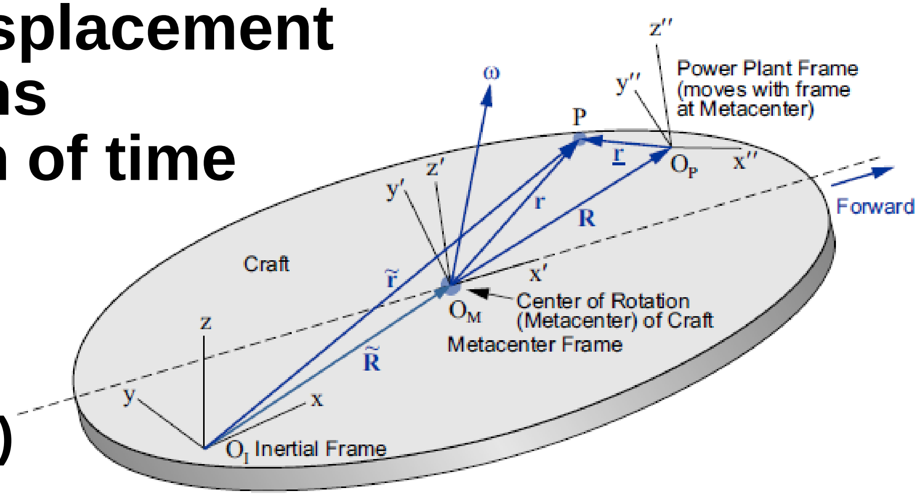
Rotational and Translational Input

- **Card 190 specifies type of transient rotational angle**
  - **Euler angles or pitch-yaw-roll angles**
- **Cards 191 – 193 specify transient rotational angles as a function of time**
  - **Amplitude (A)**
  - **Period (B)**
  - **Phase angle (C)**
  - **Offset angle (D)**
  - **Constant part of rotational speed (E)**
  - **E.G. γ = A sin (2π[t/B + C/360]) + D + Et**



Rotational and Translational Input

- **Cards 194 – 196 specify displacement in the x-, y-, and z-directions (respectively) as a function of time**
  - **Amplitude (A)**
  - **Period (B)**
  - **Phase angle (C)**
  - **E.G. $y = A \sin(2\pi(t/B + C/360])$**



- **Cards 2090NXXX specify translational displacement and rotational angle tables as a function of time**
  - **This information is added to information already entered on Cards 191 – 196**

Hydrodynamic Input

- Cards 120 – 129 specify a reference volume in the hydrodynamic system (one card for each system)
  - Each card also allows specification of x, y, and z coordinates of the reference volume relative to fixed (inertial, world) x-, y-, and z-axes
  - Any rotation is assumed to be about the origin implied by the reference volume
  - The origin is the initial metacenter of the craft
- Hydrodynamic component-specific cards allow input of position change along fixed (world, inertial) x-, y-, and z-axes due to traverse from inlet to outlet along the local x-, y-, z-axes
  - There are 9 combinations (xx, yx, zx, xy, yy, zy, xz, yz, zz)

Hydrodynamic Input (continued)

- These 9 combinations are used to obtain loop closure in the x, y, and z coordinate directions
  - Loop closure is required in all three directions for MOVING problems
  - Failure to close in an initially horizontal direction could cause a failure to close in a vertical direction after rotation
- FIXED problems are easier to input than MOVING problems
  - They require loops to close in the z-direction only

Summary

- RELAP5-3D has a moving system capability
  - Model fixed or moving reactors that may undergo acceleration due to translation and/or rotation
- The acceleration affects the momentum equation as an effective gravitation acceleration vector
- RELAP5-3D has two sets of rotation angles
  - Pitch/yaw/roll and Eulerian
- There are numerous input options for specifying the reference location and time-dependent data