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The Effect of Nodalization on the Accuracy of the Finite-Difference Solution of the Transient Conduction Equation

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Abstract

One of the important phenomena that thermal-hydraulic codes such as RELAP5 must accurately calculate is heat transfer between a fluid and solid. Currently all thermal-hydraulic safety codes use the finite-difference technique to solve the transient conduction equation. This paper will examine the effect of different nodalization strategies on the accuracy of the finite-difference solution of a transient conduction problem with one convective boundary condition and no internal heat generation. The paper concludes with recommendations for choosing an appropriate nodalization scheme for modeling conduction in a wall without internal heat generation.

Background

Transient heat transfer between a fluid and a wall plays an important role in many analyses of interest to the plant analysis and safety communities. An inaccurate calculation of this heat transfer can lead to significant distortions in the predicted plant response, most notably pressure. There are several classes of problems for which the magnitude of the heat transferred from/to an unheated surface is very important. Examples of these include modeling the condensation on the Core Makeup Tank walls for the AP600, modeling the pressurizer behavior and the boiloff of water in the inlet plenum of a reactor following a Loss-of-Coolant Accident due to the stored energy associated with reactor internals. When the problems involve a phase change, boiling or condensation, the large changes in specific volume have the ability to directly impact the pressure response of the system. Furthermore, since these heat transfer regimes are characterized by very large heat transfer coefficients, the correct determination of the wall surface temperature is required to determine the proper heat transfer. Since the accuracy of system pressure and fluid energy predictions depend on the accuracy of the transient conduction solution, it is necessary to understand which parameters can effect the accuracy of the transient conduction solution.

All of the current safety codes (e.g. RELAP5-3D [1] and TRAC-PF1 [2]), use a finite-difference technique to solve for the temperature profile in heat structures. The finite-difference technique [3] divides the domain into discrete volumes or nodes and solves for the average temperature in each node. An important aspect affecting the accuracy of the finite-difference technique is the discretization of the problem domain. When large nodes are used, the difference between the average node temperature and the temperature at the node boundary can be significant. For problems involving a fluid-metal boundary, where the wall surface temperature is the driving temperature for the heat transfer into or out of the metal, the use of an average temperature instead of the true surface temperature can lead to significant distortions in both the total heat transferred to the wall and the temperature profile within the wall. This effect has been shown schematically in Figure 1. This figure shows a comparison between an exact solution and a finite-difference representation near the surface of a wall which is convecting heat from a hot fluid. This figure illustrates the case where there is a significant discrepancy between the true surface temperature and the calculated surface node temperature.

The modeling of transient conduction equation for materials with internal heat generation (i.e. nuclear fuel) is more complex and is not included in the present study.

Analytical Solution

To provide a basis to compare the different nodalization strategies a benchmark is required. For this study, an analytical solution of a generic conduction problem is used. There are several types of transient conduction problems for which there are tractable analytical solutions. The easiest of these to examine is transient conduction in a slab which is initially isothermal. The boundary conditions which are used are a convective boundary condition with a step change in the fluid temperature on one side of the slab and an adiabatic condition on the other side. The solution to this problem [4] is represented by an infinite series and is given by:

$$\frac{T(x,t) - T_\infty}{T_0 - T_\infty} = \sum_{m=1}^{\infty} A_m e^{-\delta_m^2 \mathrm{Fo}} \cos[(1-\beta)\delta_m]$$

where the characteristic values, δ_m, are determined using the following transcendental equation;

$$\delta_m \tan(\delta_m) = \mathrm{Bi}$$

The coefficients, A_m, are given by;

$$A_m = \frac{2\sin(\delta_m)}{\delta_m + \sin(\delta_m)\cos(\delta_m)}$$

and the remaining terms are

$$\mathrm{Fo} = \text{Fourier Number} = \frac{\alpha t}{L^2}$$

$$\mathrm{Bi} = \text{Biot Number} = \frac{hL}{k}$$

$$\text{and, } \beta = \frac{x}{L}$$

It can be seen that there are only three parameters which effect the transient temperature distribution: Biot number, Fourier number and distance into the wall. The Fourier number is used to scale the time. For a given time the temperature distribution is independent of the Fourier number. The important parameters for determining the spatial temperature distribution, which in turn determine the heat transfer into the wall, are the location, β, and the Biot number, Bi. To account for the Biot number effect, several different heat transfer coefficients will be examined. Since the most important temperature is that at the surface (β=0), because it determines the energy deposited into the wall, this will be only location for which the analytical solution is calculated.

The analytical solution was solved using the Mathcad® computer program [5] for a range of Biot numbers. To determine the minimum number of terms required in the series, the expansion of the surface temperature with a Fourier number of zero is examined. It can be shown that this temperature requires the most number of terms in the series to provide an accurate prediction. A comparison of the solution with 120 terms and 10000 terms yielded the same result to seven digits, therefore, 120 terms are assumed to be sufficient.

Finite-Difference Solution

To examine the effects of the different nodalization strategies, the same problem as described above was solved using the RELAP5-3D code. To reproduce the boundary conditions which were used in the analytical problem, a constant heat transfer coefficient and constant fluid sink temperature are required. The user option was chosen to allow a constant heat transfer coefficient to be input. The constant sink temperature was achieved using the combination of a large fluid volume and large fluid velocity. This combination results in insufficient heat transfer to the fluid to increase its temperature thereby creating a constant sink temperature.

The process of determining the location for the temperature mesh points can vary from code to code. The tests which have been performed are sufficiently generic such that the same strategies can be used for a number of thermal-hydraulic codes. Three separate nodalization strategies are used in this study. They are depicted in Figure 2. The base noding uses a regular mesh where the distance between the nodes is held constant at 25% of the total wall thickness. This results in a total of five mesh points for which temperatures are calculated. The boxes in Figure 2 represent the volume of the wall associated with each mesh point. As can be seen, for a regular mesh, the surface nodes contain only one-half of the volume of the interior nodes. This is due to the manner in which the boundary conditions are handled for the surface nodes. The smaller nodes at the surface are desirable in that they will have a smaller thermal inertia than the interior nodes and will therefore respond more quickly.

The variable noding strategy in Figure 2 uses the same number of mesh points, but has a finer spacing near the surface which has the convective boundary condition and a coarser spacing near the insulated surface. The distances between successive mesh points is 10%, 20%, 30% and 40% respectively. This approach has two advantages; first, it reduces the thermal inertia associated with the surface node. As stated previously, a reduction in the thermal inertia of the surface node provides a more accurate calculation of the surface temperature and hence heat transfer to the wall. Second, this strategy allows the steep temperature gradients which occur near the wall to be better simulated.

The final nodalization strategy uses a variable mesh with more points. This nodalization is formed by dividing each of the spans used in the variable nodalization in half. For clarity, the added mesh points are shown as triangles.

The conditions which have been used in the problem are stated in Table 1. These properties are typical values for a thick steel wall. The Biot number range for this problem corresponds to a range of heat transfer coefficients of 400-40000 watt/m^2/K (72-7200 BTU/hr/ft^2/F). This range covers the phenomena of turbulent natural convection in water, single-phase convection to water or steam/air (high Reynolds numbers), condensation and boiling.

There are two formulations of the finite-difference representation of the transient conduction equation: the explicit and implicit. For the current problem with no internal heat generation, the difference between the implicit and explicit methods is whether new time (implicit) or old time (explicit) terms are used in the temperature gradient. Due to the numerical stability limitations associated with the explicit formulation, this method is not employed in current thermal-hydraulic codes. Instead the implicit formulation is used. Therefore, this study is performed with, and is appropriate for, the implicit formulation. Furthermore, the modeling guidelines which are recommended should not be confused with the numerical stability limits associated with the explicit finite-difference methodology.

Results

Since the primary impact that the transient wall conduction has on a thermal-hydraulic analysis code is as a heat sink or source, the primary variable of interest in this study is the total heat transferred to the wall. This parameter is calculated as

$$Q_T(t) = \int_0^t q''(\tau)d\tau = h\int_0^t (T_{surf}(\tau) - T_\infty)d\tau$$

For each of the Biot numbers examined, the analytical solution is compared to the finite-difference calculation performed with each of the noding strategies. The analytical solution is accompanied by 20% error bands. If the predictions are within the error bands, the noding strategy is deemed to be sufficient. Since the inaccuracies in the finite-difference calculations are largest at the transient initiation, only the first 50 seconds of transient is analyzed. For this problem, 50 seconds represents a Fourier number of 0.04.

<u>Biot Number = 1</u>

This case represents the easiest problem for finite-difference methodology since the heat transfer is gradual and steep temperature gradients are not encountered in the problem. The heat transfer coefficient associated with this scenario is approximately 400 watt/m^2/K (~70 BTU/hr/ft^2/F).

This would correspond to either very turbulent natural convection in water or forced convection. As seen in Figure 3, the base noding accurately represents the total heat transferred to the wall. The corresponding plot for the surface temperature is provided in Figure 4. From this figure it can be seen that surface temperature response for the base noding is not well predicted; however, since the heat transfer coefficient is low, the error in the surface temperature prediction does not translate into a large effect on the total heat transferred to the wall.

Biot Number = 5

The heat transfer coefficient associated with this scenario is approximately 2000 watt/m^2/K (~350 BTU/hr/ft^2/F). This value is appropriate for forced convection to water. Figure 5 shows that for this case there is a much larger difference among the different nodalization strategies. Furthermore, since the base noding barely lies within the 20% error bands on the analytical solution this is the limiting Biot number for the base nodalization.

Biot Number = 15

The heat transfer coefficient associated with this scenario is approximately 6000 watt/m^2/K (~1000 BTU/hr/ft^2/F). This value is appropriate for boiling conditions. The comparison of total heat transferred is presented in Figure 6. For this problem the total heat transferred for base nodalization is outside the 20% error bands for most of the first 50 seconds. In fact, at 10 seconds the error is almost 40%. Figure 7 shows the surface temperature response. From this figure, the gross error of the surface temperature can easily be seen. In addition, the variable mesh prediction has deviated slightly from the analytical solution. One interesting feature of this plot is the temperature undershoot for the base noding case. This is attributed to the finer nodalization more accurately reproducing the temperature gradients near the surface, and hence the heat conducted toward the insulated boundary condition.

Biot Number = 50

The heat transfer coefficient associated with this scenario is approximately 20000 watt/m^2/K (~3600 BTU/hr/ft^2/F). This value is appropriate for boiling heat transfer. Figure 8 is a plot comparing the total heat transferred by the different nodalizations to the analytical solution. For this problem, the predicted total heat transfer for the base case is almost twice the analytically determined value at 5 seconds. For this problem, the variable mesh strategy stays within the 20% error bands except for very early times in the transient (time less than 5 seconds).

Biot Number = 100

This problem is the most challenging for the finite-difference technique. The heat transfer coefficient for this problem is approximately 40000 watt/m^2/K (~7200 BTU/hr/ft^2/F) which is consistent with boiling or condensation heat transfer. This value is most representative for modeling condensation on the pressurizer walls. For this problem, only the refined variable mesh produces results that are within the 20% error bands. At 5 seconds, the base noding predicts the total heat transfer to be greater than twice the analytical solution and the error in the variable noding is approximately 25%. At 2.5 seconds the base case predicts greater than 2.5 times the analytically determined total heat transfer.

Suggested Guidelines

It is clear that the nodalization strategy which is chosen can have a significant impact on accuracy of the transient conduction solution.

Since the performance of the different nodalization strategies is a strong function of Biot number, an analyst should consider what heat transfer mechanisms (i.e. convection, boiling and/or condensation) will be appropriate for any given structure. Based on this determination, an appropriate heat transfer coefficients and Biot number must be calculated. Based on the calculated Biot number, the following guidelines are recommended:

Variable Mesh

For expected Biot Numbers larger than 15, a graduated, or variable, mesh should be used. The use of such a mesh allows for an accurate calculation of the temperature gradients with a smaller number of nodes than are required for a uniform mesh.

Minimum Mesh Size

Since the Biot Number has a significant impact on the maximum allowable size of the surface node, a relationship between the two is desired. To determine this relationship, the largest acceptable relative mesh size for the surface node (β_{max}) was plotted for each Biot Number (Figure 11). The best relationship was found to be between the inverse of β_{max} and the Biot number.

The β_{max} values were determined by calculating the fraction of the wall contained in the surface for the coarsest mesh which met the accuracy criteria for each case. This fractional volume represents all of the wall which is included in the surface node which is one-half of the relative depth of the first interior mesh point.

The line in Figure 11 represents the recommended maximum relative surface node size. Since only three different meshes were examined, the resolution is very coarse. Only the data points at Biot numbers of 5, 50 and 100 were used to determine equation for the line representing the recommended surface node size. The other data points were omitted since for each case there was a larger Biot number for which the allowable β_{max} was the same. The equation for the recommended maximum surface node size is given by

$$\frac{1}{\beta_{max}} = 0.338\mathrm{Bi} + 5.2$$

It is interesting to compare this recommendation to a lumped capacitance model. For very low Biot numbers (i.e <0.1) the lumped parameter model which only uses one characteristic temperature for a wall is appropriate. Using the recommended guidelines the appropriate β_{max} for case of a Biot number of zero is 19%. A uniform mesh with three nodes results in a β of 25% for the surface node, which is only slightly larger than the recommended value. Therefore at very low Biot numbers these guidelines would recommend three mesh points as opposed to the one required in the lumped capacitance model.

Conclusions

This study examined the effect of different nodalization strategies on the accuracy of the finite-difference solution of the transient conduction equation. By comparing different nodalization strategies to analytical solutions, the error associated with each strategy was assessed. For the primary variable of interest, the total heat transferred to the wall, the base nodalization strategy predicted a total heat flow which at times was more than 2.5 times the analytical value. Based on these comparisons, guidelines for modeling transient conduction in unheated walls for a large range of surface heat transfer coefficients have been recommended.

References

1. Idaho National Engineering and Environmental Laboratory, *RELAP5-3D© Code Manual, Volumes 1-5*, INEEL-EXT-98-00834, 1999.
2. Los Alamos National Laboratory, *TRAC-PF1/MOD1: An Advanced Best-Estimate Computer Program for Pressurizer Water Reactor Thermal-Hydraulic Analysis,* NUREG/CR-3858, 1986.
3. Incropera, F. P. and D. P. DeWitt, *Fundamentals of Heat Transfer,* John Wiley & Sons, New York, 1981.
4. Kreith, F., *Principles of Heat Transfer,* International Textbook Company, Scranton, 1961.
5. *Mathcad User 's Guide,* MathSoft Inc., 1999.

Nomenclature

Item	Definition	Brit. Units	SI Units
h	heat transfer coefficient	BTU/ hr/ft^2/ F	watt/ m^2/K
k	thermal conductivity	BTU/ hr/ft/F	watt/ m/K
q’’	heat flux	BTU/ hr/ft^2	watt/ m^2
t	time	hr or sec	sec
x	distance	ft	m
C_p	specific heat	BTU/ lb/F	Joule/ K
Bi	Biot number	none	none
Fo	Fourier number	none	none
L	wall thickness	ft	m
Q_T	total heat flow per unit area	BTU/ ft^2	Joule/ m^2
T(x,t)	local temperature	F	K
T_0	initial temperature	F	K
T_∞	sink temperature	F	K
α	thermal diffusivity	ft^2/ sec	m^2/ sec
β	non-dim. distance	none	none
ρ	density	lb/ft^3	kg/m^3

Table 1: Input Parameters

Parameter	Value
k	0.01 BTU/sec/ft/F 62.3 watt/m/K
ρC_p	50 BTU/ft^3/F 3.53 10^6 joule/m^3/K
L	0.5 ft 0.152 m
α	2.00 10^{-4} ft^2/sec 1.86 10^{-5} m^2/sec
T_0	605.0 F 318.3 C
T_∞	600.0 F 315.6 C

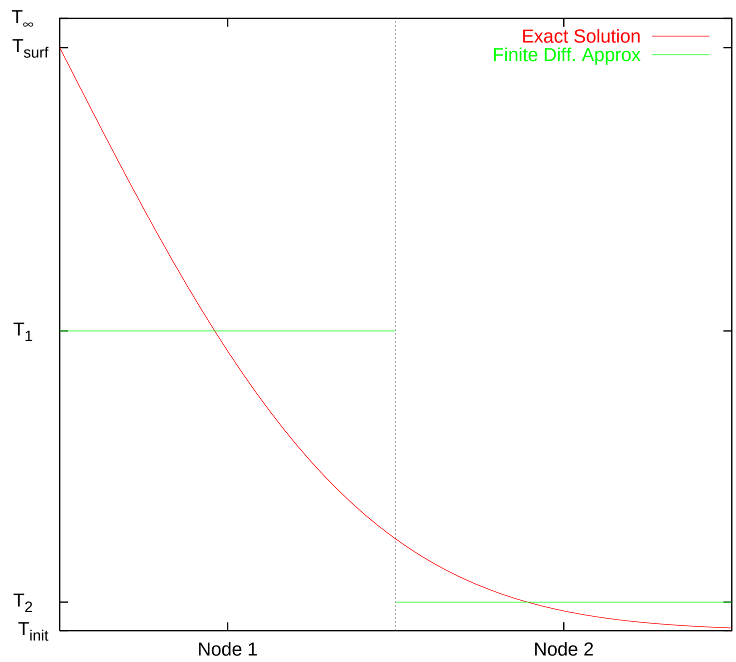

Figure 1 : Comparison of Finite-Difference Approximation to the Exact Solution

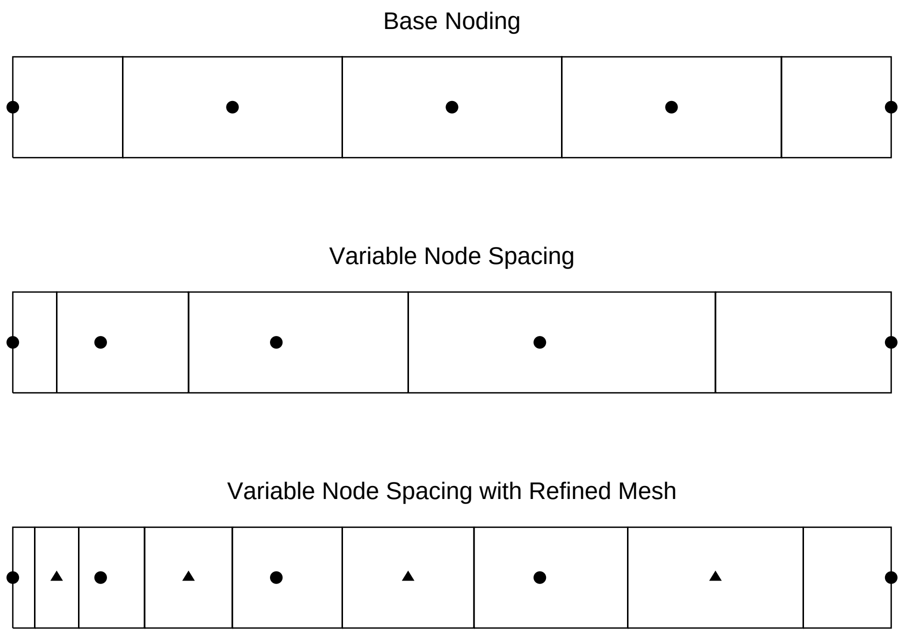

Figure 2 : Schematic of the Different Nodalization Strategies Examined

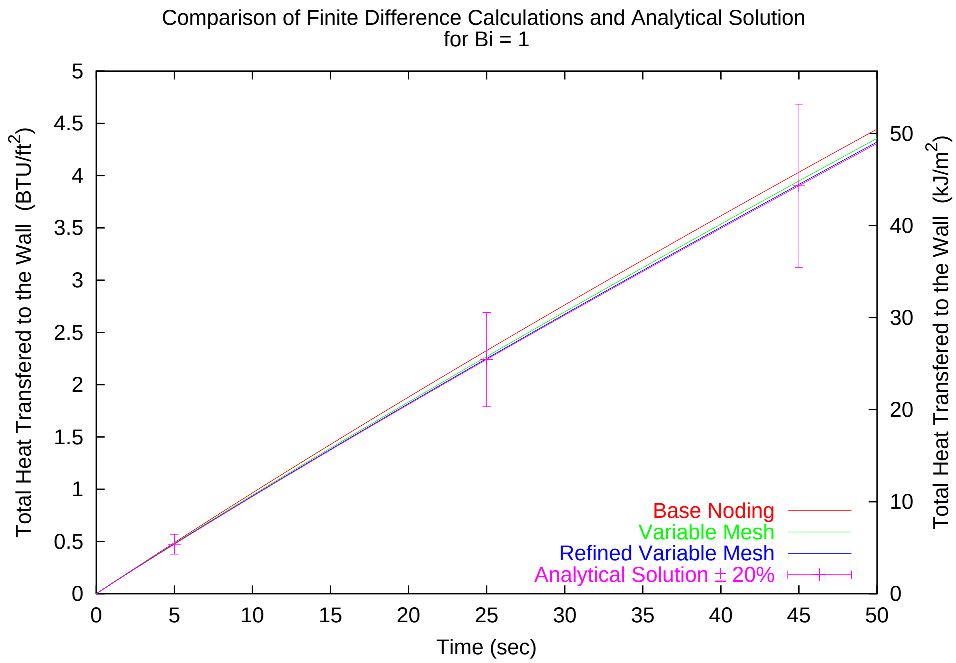

Figure 3 : Total Heat Flow for Bi = 1

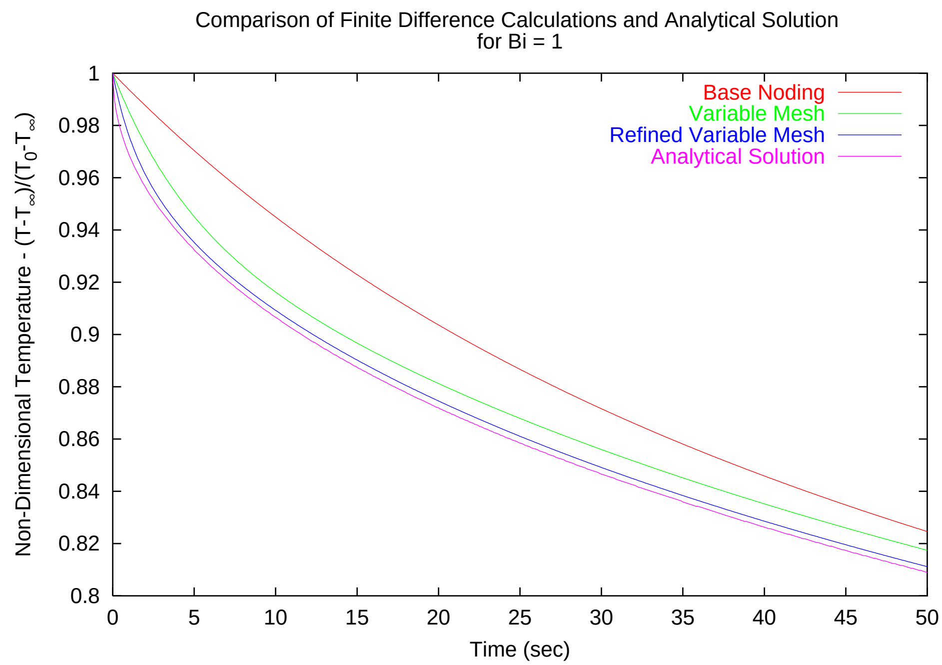

Figure 4 : Non-Dimensional Surface Temperature for Bi = 1

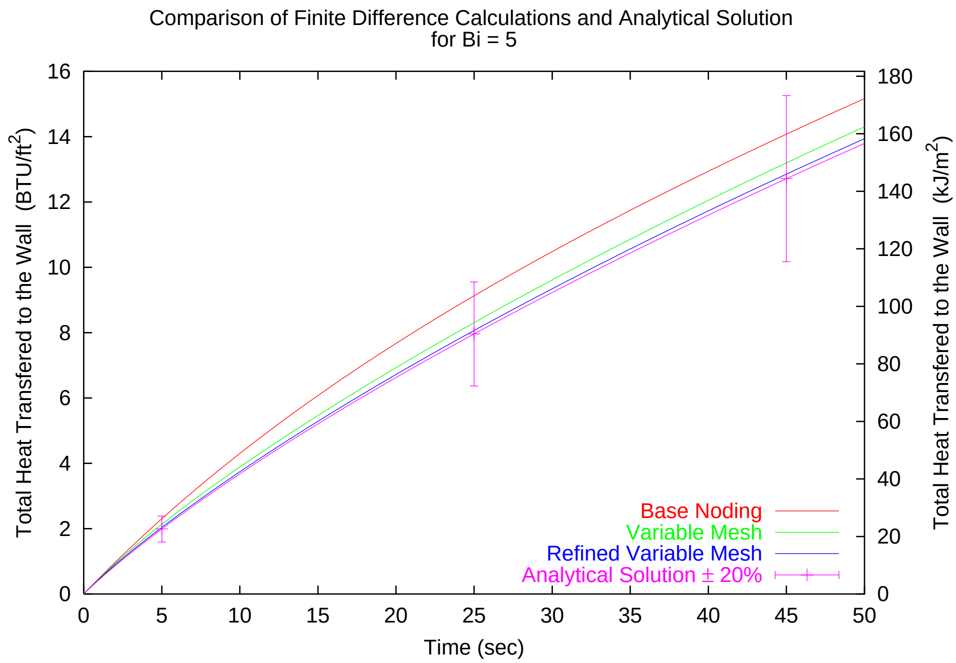

Figure 5 : Total Heat Flow for Bi = 5

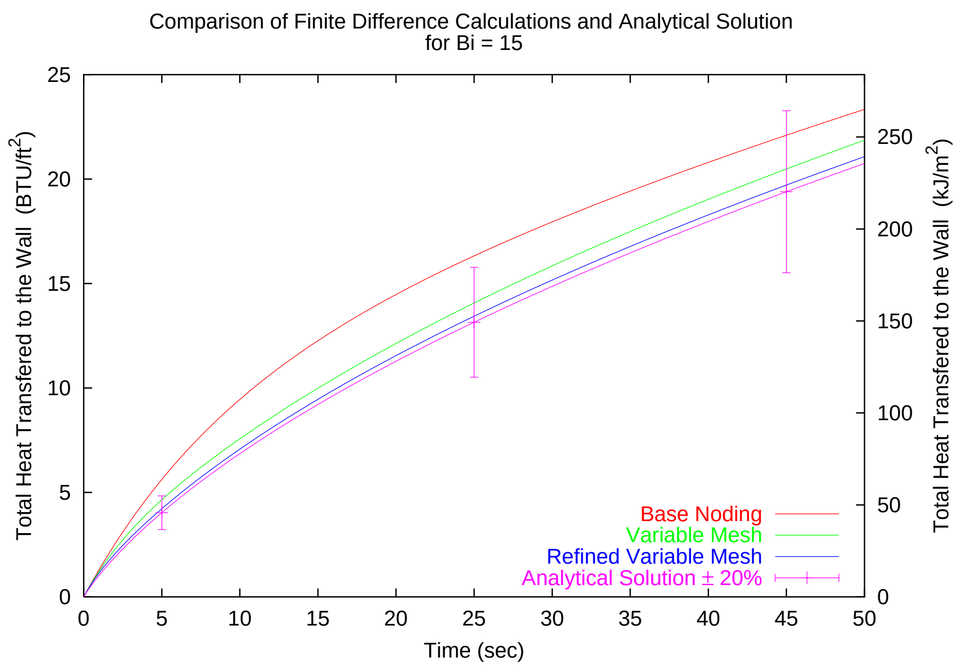

Figure 6 : Total Heat Flow for Bi = 15

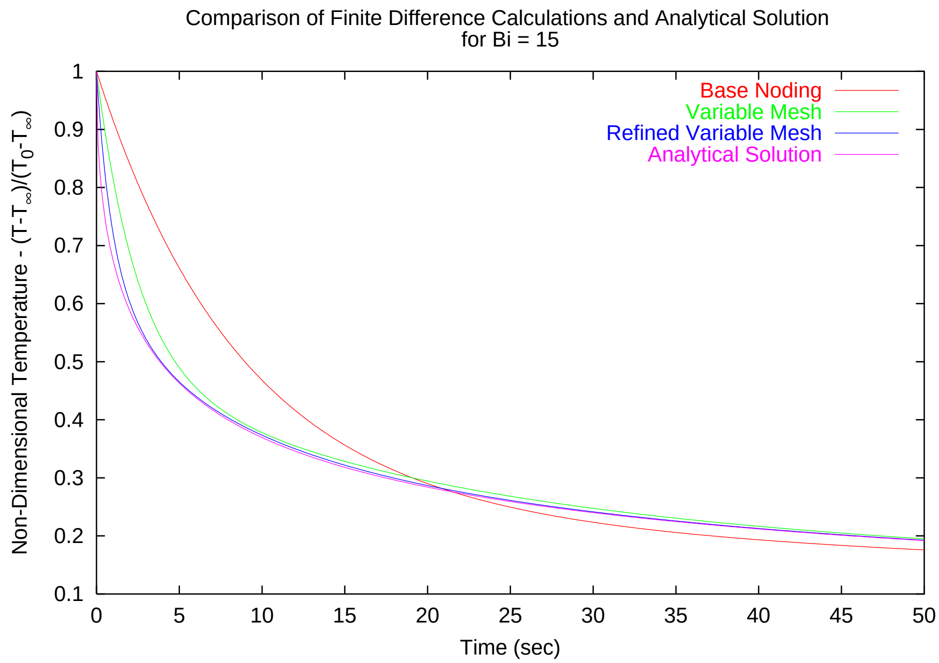

Figure 7 : Non-Dimensional Surface Temperature for Bi = 15

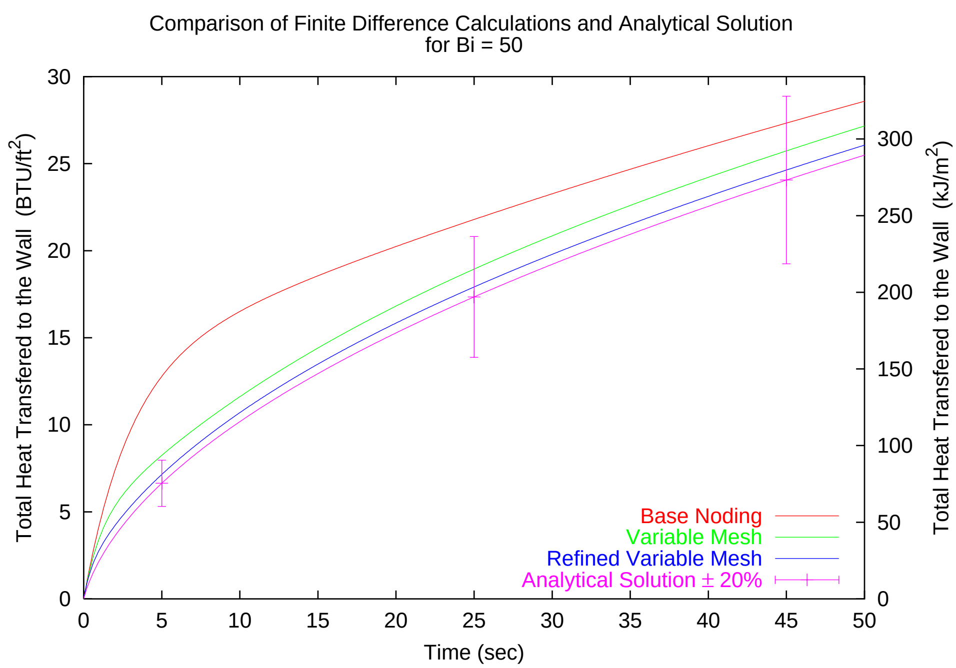

Figure 8 : Total Heat Flow for Bi = 50

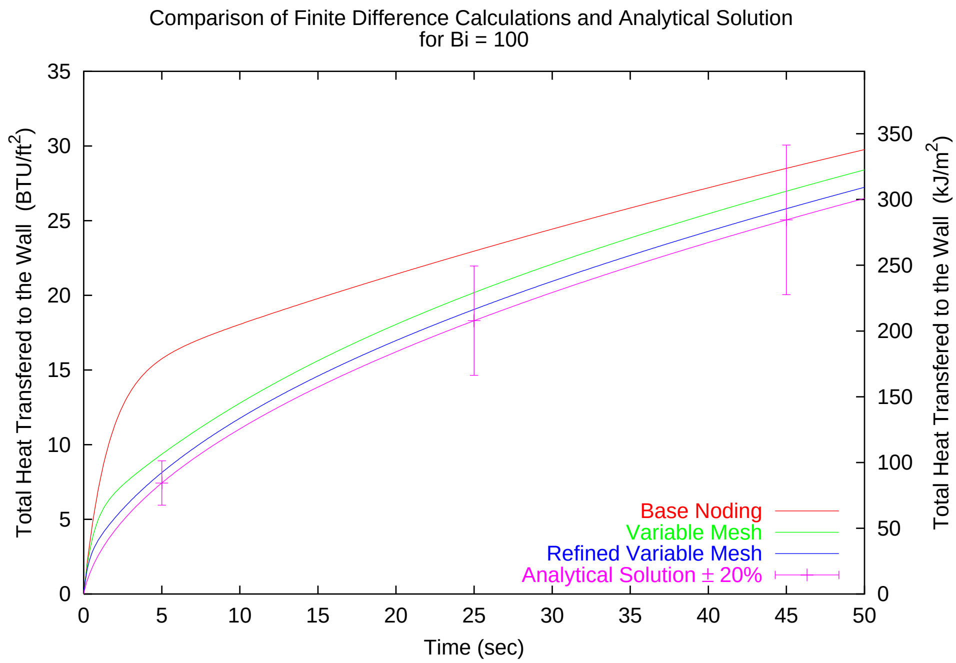

Figure 9 : Total Heat Flow for Bi = 100

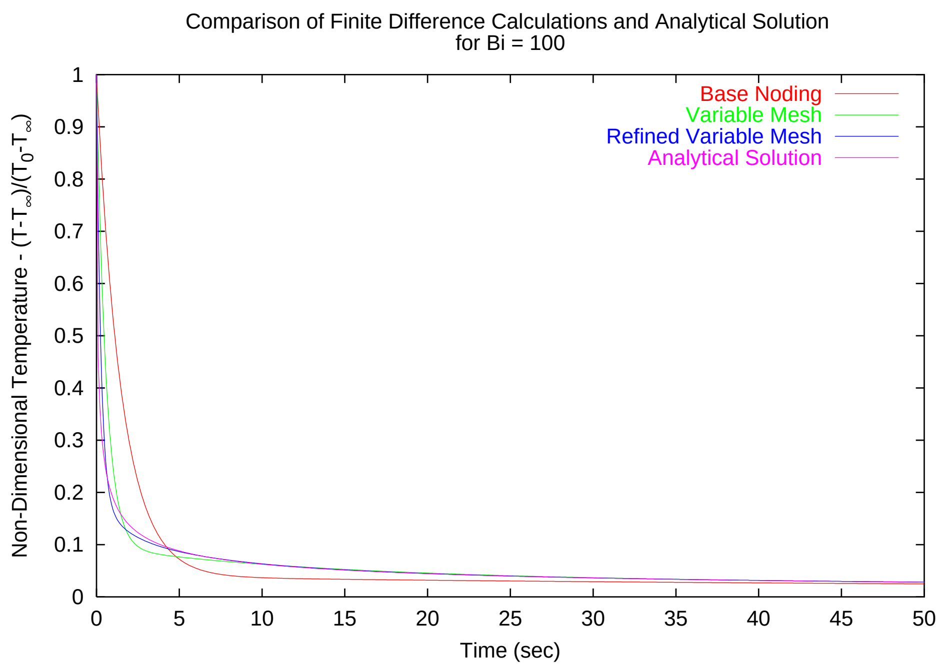

Figure 10 : Non-Dimensional Surface Temperature for Bi = 100

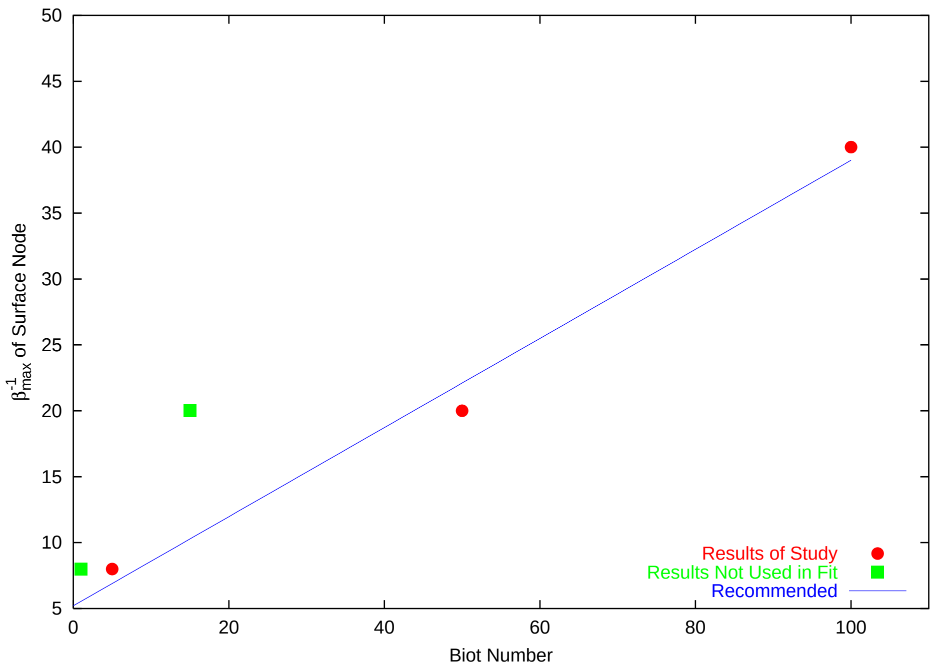

Figure 11 : Effect of Biot Number on Suggested Surface Node Size